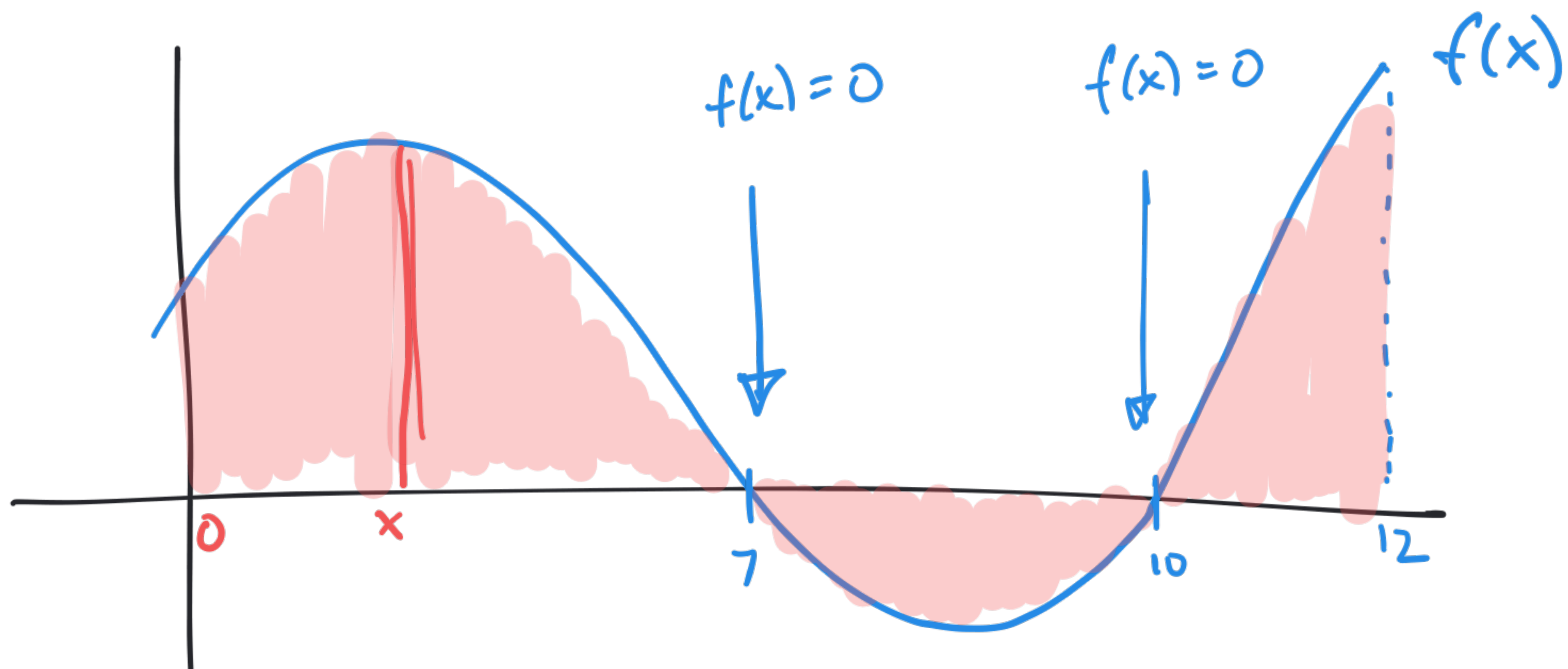
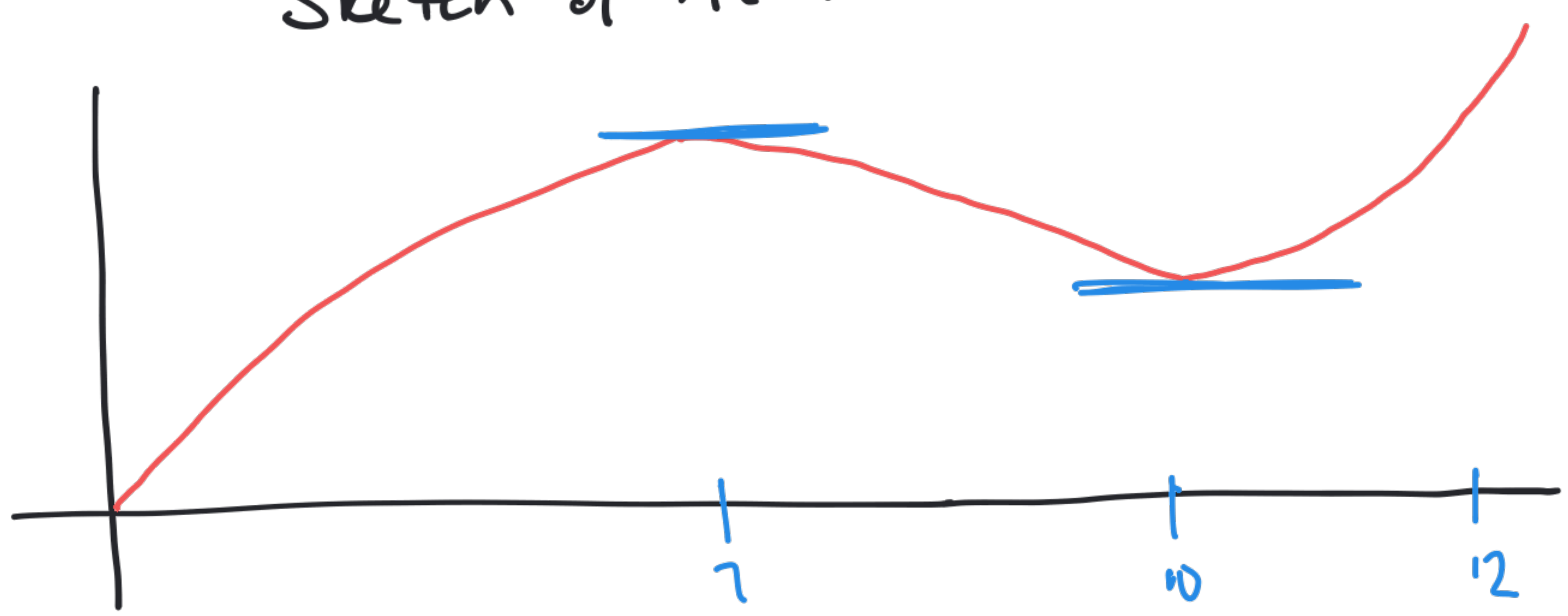
Intro Video: Section 5.3 part 1
The fundamental theorem of calculus, part 1

Math F251X: Calculus I



$$A(x) = \int_0^x f(t)\,dt$$

Sketch of A(x)



The fundamental theorem of calculus, part 1:

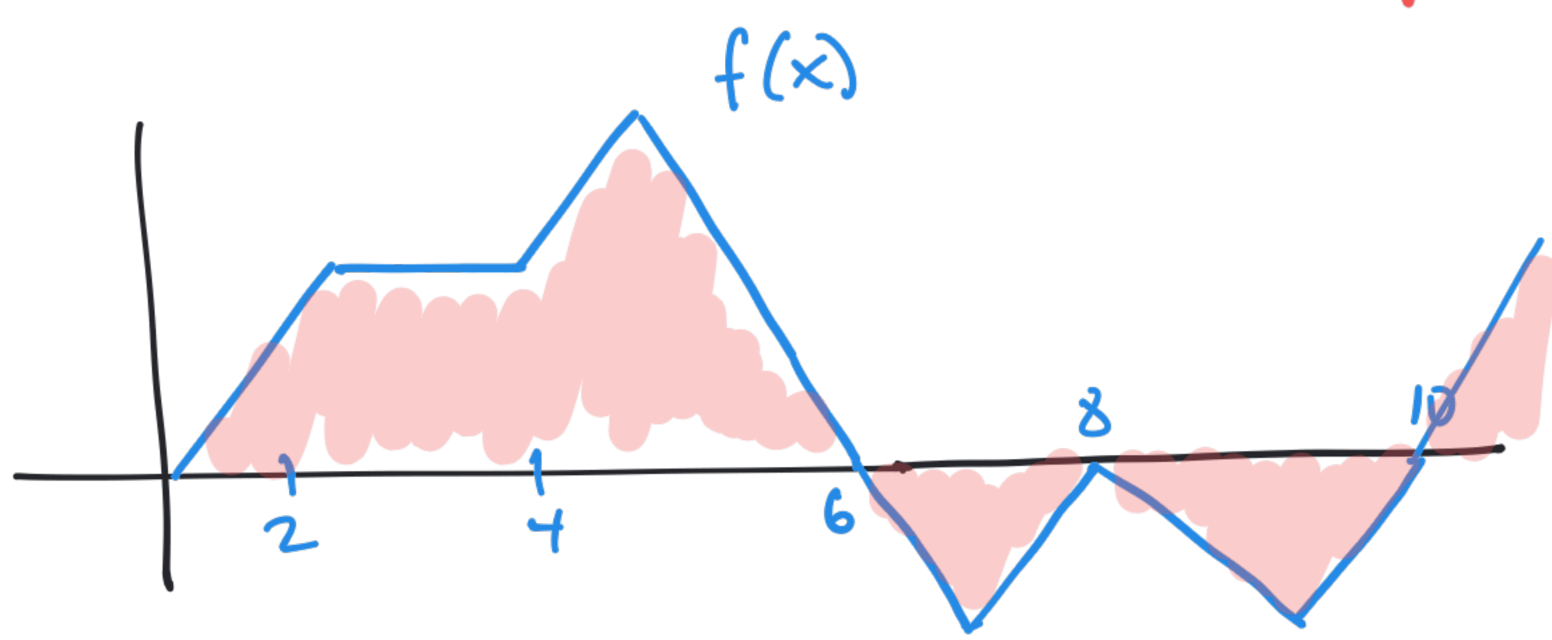
If f is continuous on $[a,b]$, then the function

$$g(x) := \int_a^x f(t)\,dt$$

is continuous on $[a,b]$, differentiable on (a,b), and

$$g'(x) = f(x).$$

Note f does not need to be differentiable!



$g(x) = \int_0^x f(t)\,dt$

local max at $x = 6$

local min at $x = 10$

Example

The "integrand"

① Let $g(x) = \int_4^x \boxed{\sqrt{t + t^3}}\, dt$. What is $g'(x)$?

$$g'(x) = \sqrt{x + x^3}$$

② Let $F(t) = \int_t^3 1 + \tan(x)\, dx = -\int_3^t 1 + \tan(x)\, dx = \int_3^t -(1 + \tan(x))\, dx$

$$F'(t) = -(1 + \tan(x))$$

Example:

$h(x) = \int_1^{\boxed{e^x}} \cos(t)\,dt$. What is $h'(x)$?

FTC 1 says $g(x) = \int_a^x f(t)\,dt \Rightarrow g'(x) = f(x)$

Let $u = e^x$, and let $j(x) = \int_1^x \cos(t)\,dt$.

Then $h(x) = j(u(x))$. By the chain rule...

$$h'(x) = j'(u) \cdot \frac{du}{dx}$$

$$= \cos(u) \cdot \frac{du}{dx}.$$

$$\boxed{h'(x) = \cos(e^x) \cdot e^x}$$

Example:

$$h(x) = \int_3^{x^5} \frac{1}{1+t^2}\,dt. \quad \text{What is } h'(x)?$$

Let $u = x^5$, and $j(x) = \int_3^{x} \frac{1}{1+t^2}\,dt$, so

$$h(x) = j(u) \Longrightarrow$$

$$h'(x) = \boxed{j'(u)}\,\frac{du}{dx}$$

use FTC 1

$$= \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$= \frac{1}{1+(x^5)^2} \cdot (5x^4)$$

Example:

$$g(x) = \int_{3x}^{5x} \arctan(t)\, dt. \quad \text{What is } g'(x)?$$

Trick #1: Split up the integral by using the fact that

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

So $g(x) = \int_{3x}^{5x} \arctan(t)\,dt = \int_{3x}^{0} \arctan(t)\,dt + \int_0^{5x} \arctan(t)\,dt$

$$= -\int_0^{3x} \arctan(t)\,dt + \int_0^{5x} \arctan(t)\,dt$$

$u = 3x$, $j(x) = \int_0^x \arctan(t)\,dt$ $\qquad$ $v = 5x$, $k(x) = \int_0^x \arctan(t)\,dt$

$$g'(x) = -j'(u)\frac{du}{dx} + k'(v)\frac{dv}{dx} = -\arctan(3x)(3) + \arctan(5x)(5)$$